

# FYP Presentation

## IMAGE DEBLURRING

### Image Upsampling via Tight Frames

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# Agenda

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- Introduction

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- Tight frames

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- Multi-resolution analysis

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- Proposed Algorithm

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- Results

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- Conclusion and future directions

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- Reconstruct a 512 x 512 pixels image  $\mathbf{f}_{desired}$ .

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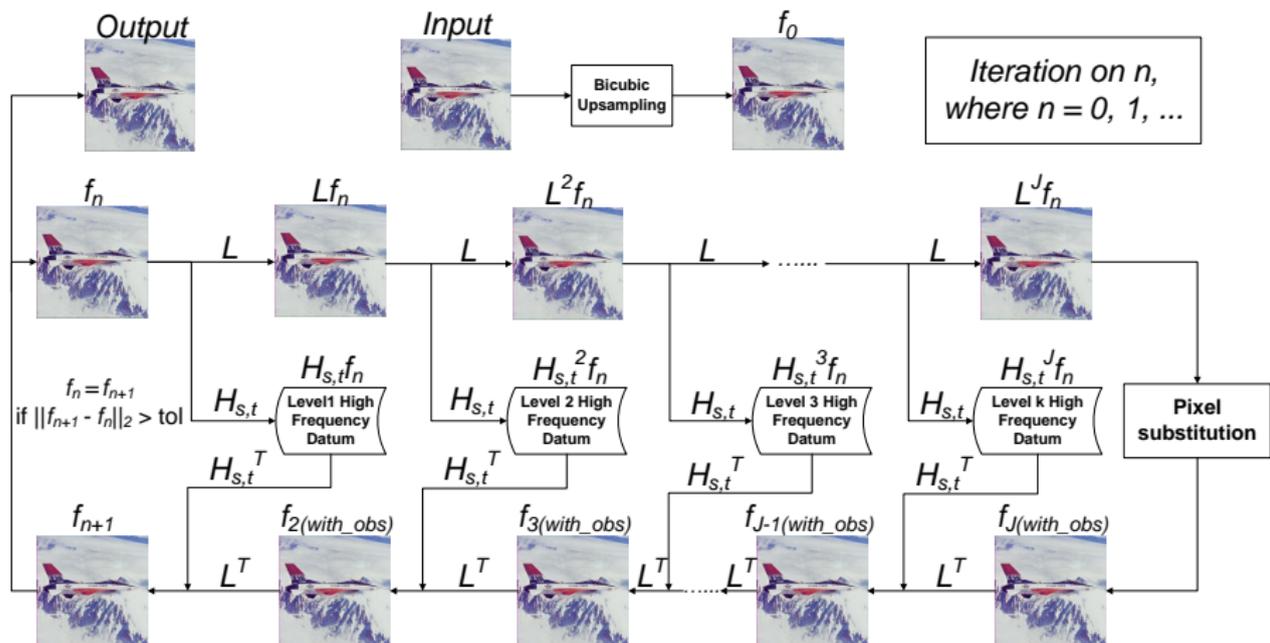
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- **Tight Frame** and **Multi-resolution Analysis**
- to design our upsampling algorithm.

# Our Proposed Algorithm



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- but not **image chrominance**.
- In addition, 2-D images are **stacked column by column** as a **1-D column vector** in our algorithm.

# Tight Frames

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A system  $X$  is called a *tight frames system* of  $L_2(\mathbb{R})$  if

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A *tight frames system* can be constructed by starting with a *scaling function*  $\phi \in L_2(\mathbb{R})$  and letting it satisfies the *dilation equation*:

$$\phi(x) = \sum_{n=-l_1}^{l_2} \sqrt{2} h_\phi(n) \phi(2x - n) \quad \forall x \in \mathbb{R}. \quad (1)$$

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With this  $\phi$  and given corresponding **high-pass filters**  $h_\psi$  with length  $(s_2 - s_1)$ ,

one can find the **tight frames function**  $\psi$  by satisfying a similar **dilation equation**

$$\psi(x) = \sum_{n=-s_1}^{s_2} \sqrt{2} h_\psi(n) \phi(2x - n) \quad \forall x \in \mathbb{R}. \quad (2)$$

Hence, given suitable **low-pass** and **high-pass filters**, one can construct a **tight frames system**

$$X_{\Psi} := \left\{ 2^{k/2} \psi(2^k x - j) \mid \psi \in \Psi; k, j \in \mathbb{Z} \right\}.$$

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- **low-pass**:  $h_0 = \frac{1}{4} [1, 2, 1]$ ;
- **high-pass**:  $h_1 = \frac{\sqrt{2}}{4} [1, 0, -1]$  and  $h_2 = \frac{1}{4} [-1, 2, -1]$ .

For 2-D images, **tensor product** is used, hence the **2-D filters**  $h_{s,t}$ ,  $s, t = 0, 1, 2$ , are defined by

$$h_{s,t} = h_t \otimes h_s.$$

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e.g.  $H_0$  for  $h_0$  and  $H_{0,0}$  for  $h_{0,0}$  are written as

$$H_0 = \frac{1}{4} \begin{bmatrix} 2 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 2 & 1 & \ddots & 0 & 0 \\ 0 & 1 & 2 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 2 & 1 \\ 1 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix} \quad \text{and} \quad H_{0,0} = H_0^T \otimes H_0.$$

Then, we have this **perfect reconstruction identity**:

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Our algorithm is basically designed by using (3).

# Multi-resolution Analysis

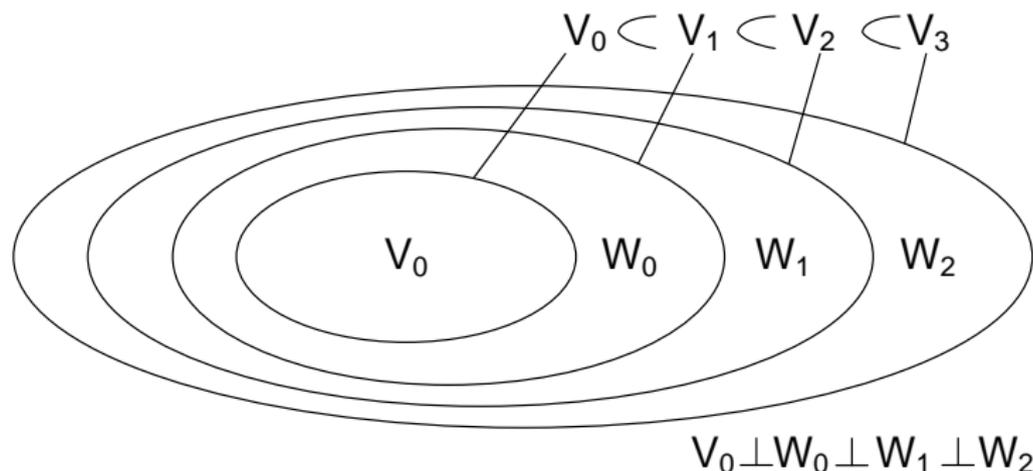
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It can be illustrated by this graph:



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- By multi-resolution analysis,

$$V_J = W_J \oplus W_{J-1} \oplus \cdots \oplus W_0 \oplus V_0.$$

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 \end{aligned} \tag{4}$$

And our proposed algorithm is iterating on this equation (4) as:

$$\begin{aligned}
 \mathbf{f}_{n+1} &= (H_{0,0}^T)^J D_{0,0}^T \mathbf{f}_{input} + (H_{0,0}^T)^J \sum_{\substack{m,n=0 \\ (m,n) \neq (0,0)}}^1 D_{m,n}^T D_{m,n} (H_{0,0})^J \mathbf{f}_n \\
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- 4 Set  $\mathbf{f}^* = \mathbf{f}_{n+1}$  if  $\|\mathbf{f}_{n+1} - \mathbf{f}_n\|_2 \leq tol$ , where  $\mathbf{f}^*$  is our upsampled image; else set  $\mathbf{f}_n = \mathbf{f}_{n+1}$  and repeat step 3 and 4.

# Results

# Results

- Click Here (Hyper Link)
- Local Files:  
->

Bicubic



TF Level 1



Bicubic



TF Level 4



Bicubic



TF Level 7



[Shan, 2009]



TF Level 1



[Shan, 2009]



TF Level 4



[Shan, 2009]



TF Level 7



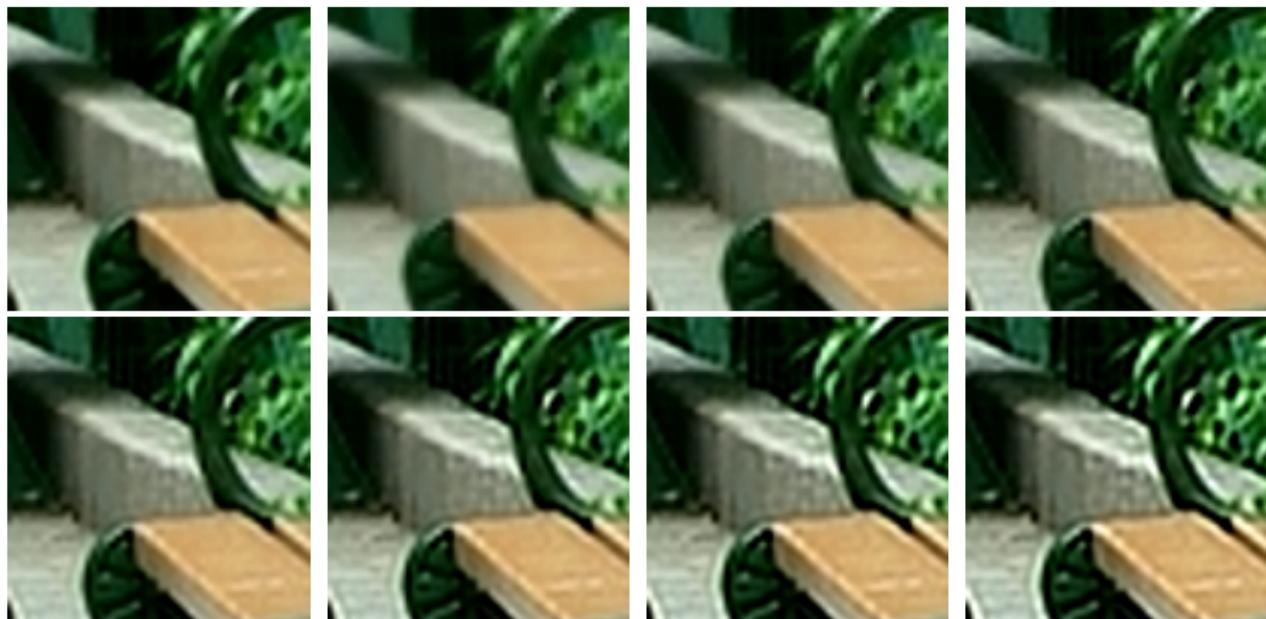


Figure:

By [Shan,2009], Level 1, Level 2 and Level 3 (Upper: From left to right)

Level 4, Level 5, Level 6 and Level 7 (Lower: From left to right)

Bicubic



TF Level 1



Bicubic



TF Level 4



Bicubic



TF Level 7



[Shan, 2009]



TF Level 1

[Shan, 2009]



TF Level 4

ERG 4920CT 09/10

[Shan, 2009]



TF Level 7

ERG 4920CT 09/10



Figure:

By [Shan,2009], Level 1, Level 2 and Level 3 (Upper: From left to right)

Level 4, Level 5, Level 6 and Level 7 (Lower: From left to right)

Bicubic



TF Level 1



Bicubic



TF Level 4



Bicubic



TF Level 7



[Shan, 2009]



TF Level 1



[Shan, 2009]



TF Level 4



[Shan, 2009]



TF Level 7



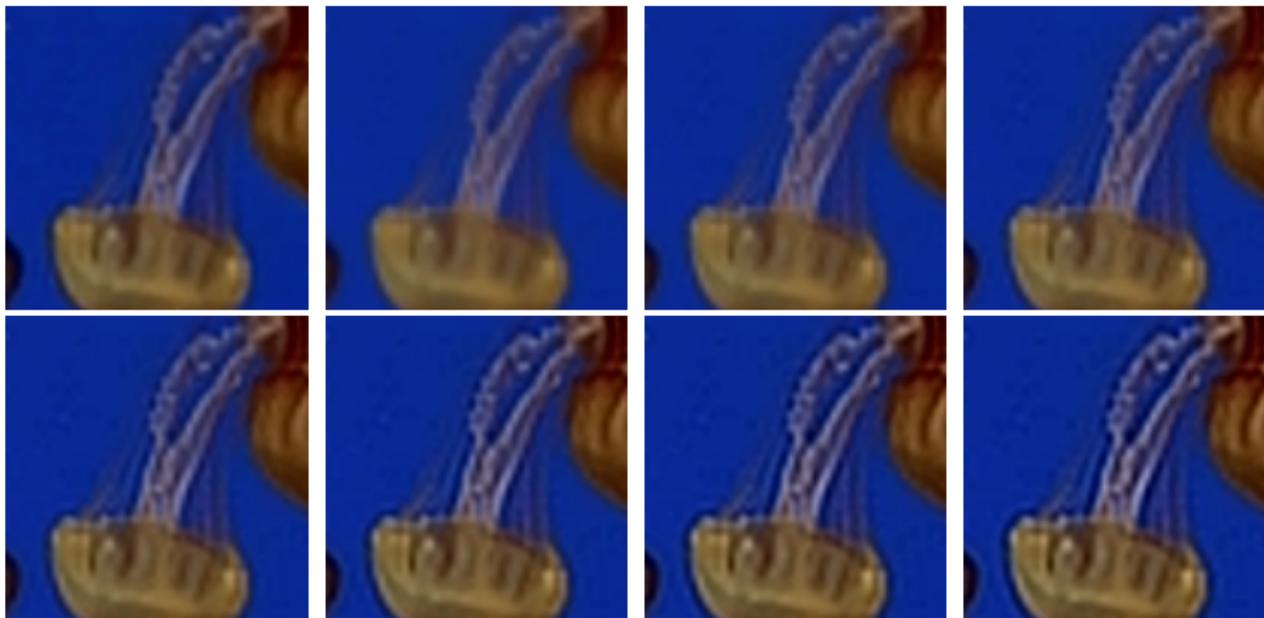


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# Thanks for your attentions!

## Q & A