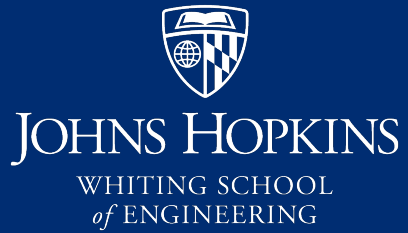


Dense Point-to-Point Correspondences Between Genus-Zero Shapes

Sing Chun Lee, Misha Kazhdan

Symposium on Geometry Processing 2019 (SGP 2019)

Outline

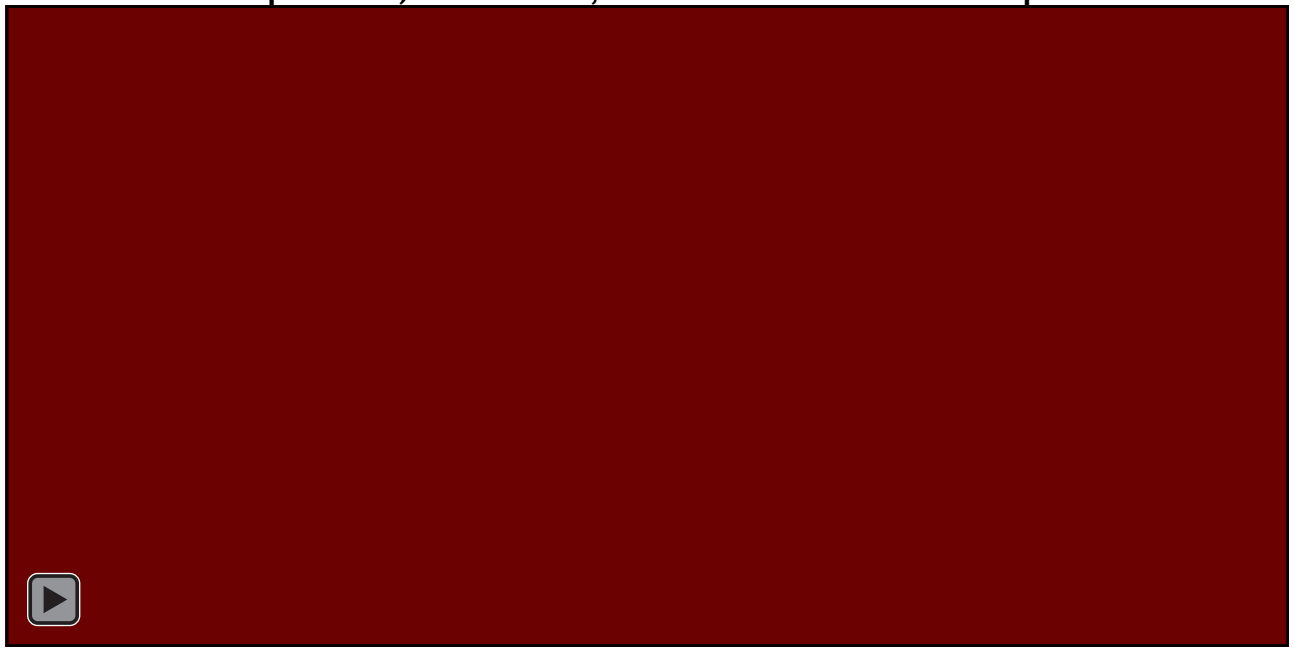
- Introduction & Related Work
- Our Approach
- Results
- Limitations & Future Work

Outline

- Introduction & Related Work
- Our Approach
- Results
- Limitations & Future Work

Dense Correspondence Between Genus-Zero Shapes

- Given genus-zero shapes $M, N \subset \mathbb{R}^3$, find a dense correspondence map



Related Work

- Rigid Registration
  - Procrustes Method [Horn, 1987]
  - Iterative Closest Points [Besl and McKay, 1992] [Rusinkiewicz and Levoy, 2001]
  - Fast Correlation [Funkhouser *et al.*, 2004]

- Isometry Invariant Embedding
  - Generalized Multi-Dimensional Scaling [Bronstein *et al.*, 2006]
  - Functional Maps [Ovsjanikov *et al.*, 2012] [Ovsjanikov *et al.*, 2017] [Litany *et al.*, 2016] [Cosmo *et al.*, 2019]

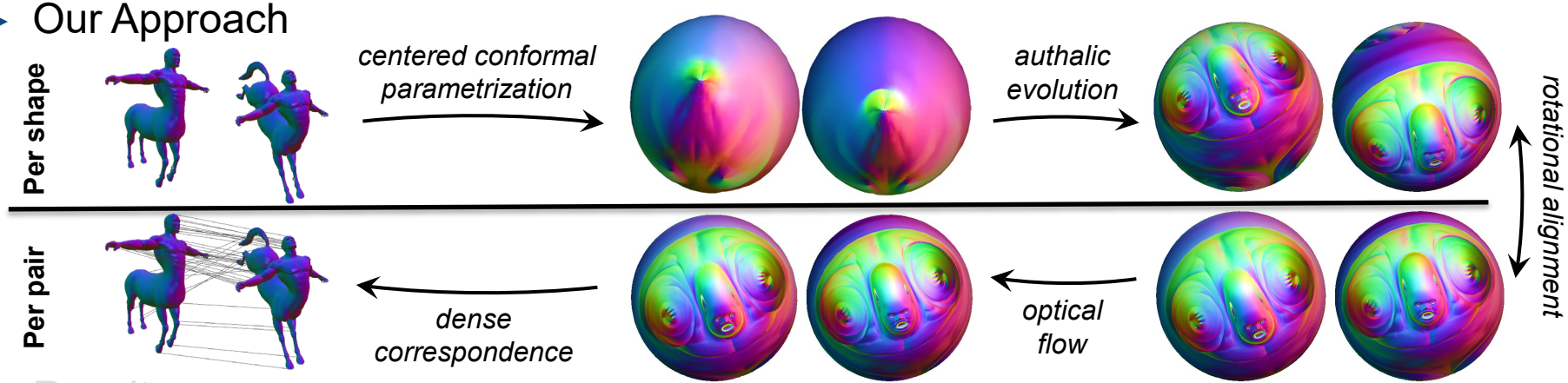
Related Work

- Map Refinement
  - Bijective Continuous ICP [Ren *et al.*, 2018]
  - Reversible Harmonic Maps [Ezuz *et al.*, 2019]

- Conformal Parameterization
  - Möbius Voting [Lipman and Funkhouser, 2009]
  - Blended Intrinsic Maps [Kim *et al.*, 2011]
  - Möbius Registration [Baden *et al.*, 2018]

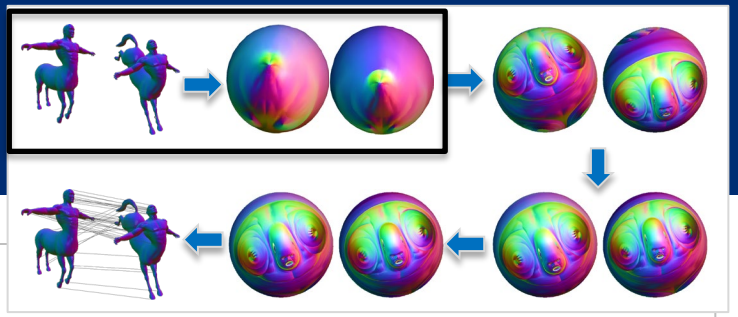
Outline

- Introduction & Related Work
- Our Approach

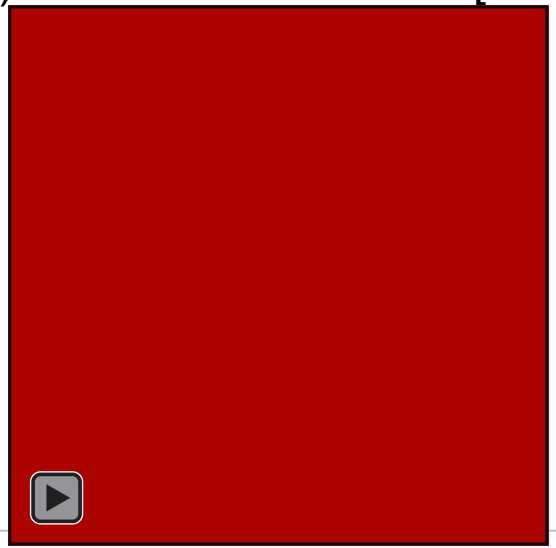


- Results
- Limitations & Future Work

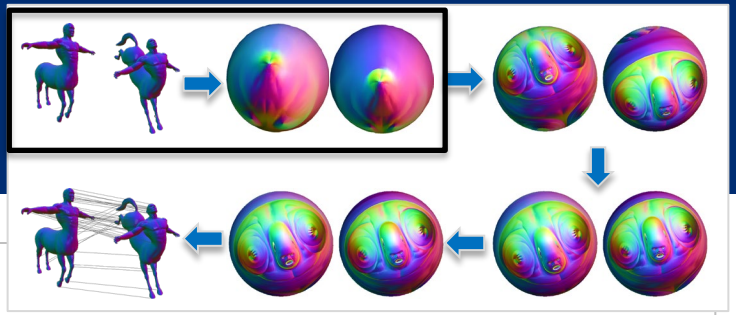
Our Approach (Per Shape)



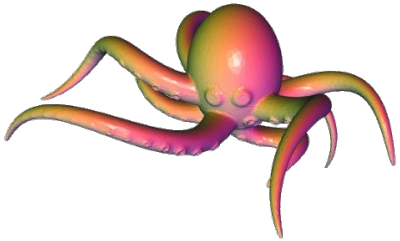
- **Conformal Parameterization**
  - Goal: Parameterize the input shape over S^2
  - Method: (Conformalized) mean curvature flow [Kazhdan *et al.*, 2012]



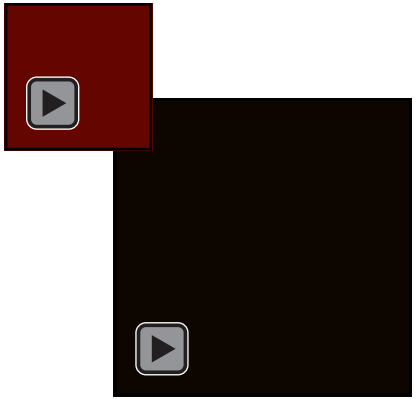
Our Approach (Per Shape)



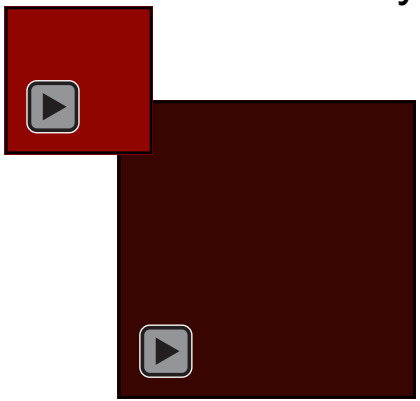
- **Conformal Parameterization**
  - Normalize for inversion by Möbius centering [Baden *et al.*, 2018]
  - Scale factors $\lambda :=$ how much surface area sits over a region of S^2
  - Approach: find the inversion making the scale factors uniformly distribute



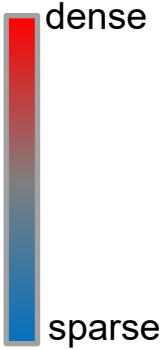
Input shape



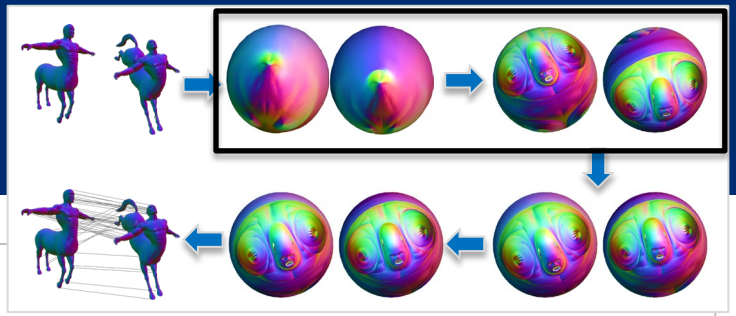
w/o centering



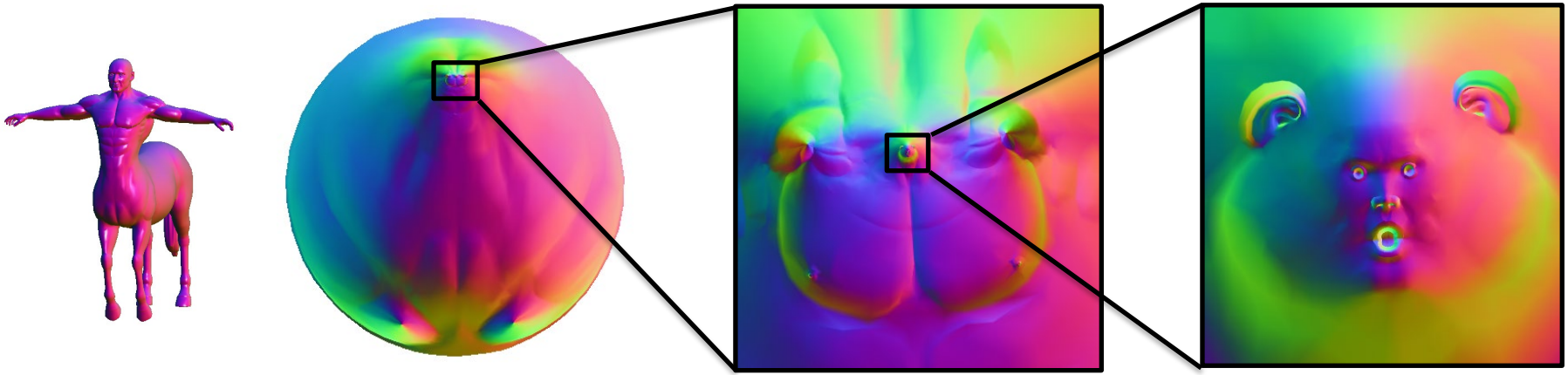
w/ centering



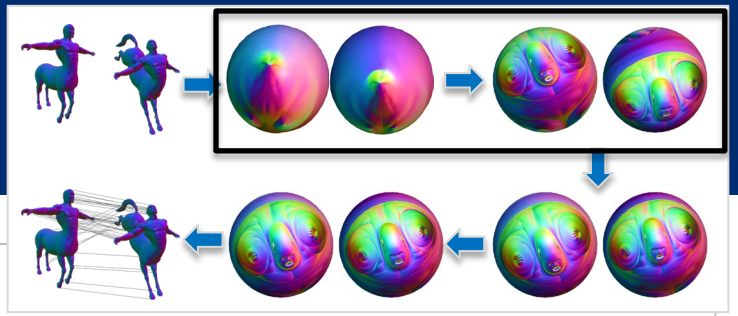
Our Approach (Per Shape)



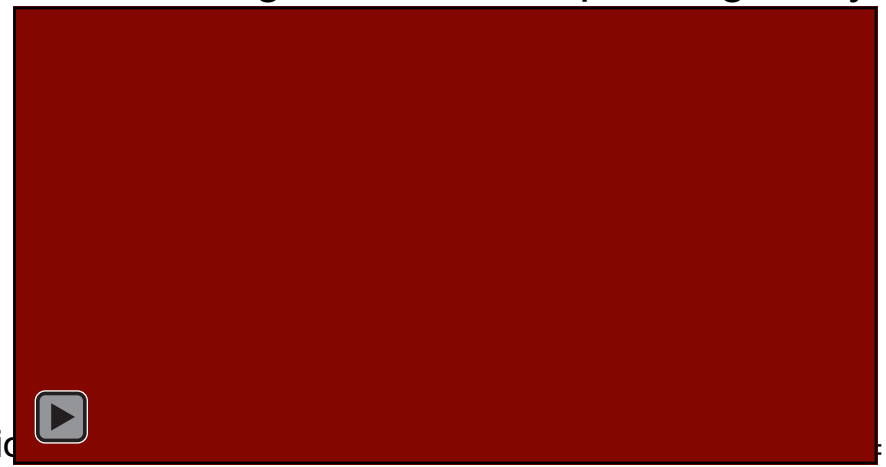
- **Authalic Evolution** [Zou *et al.*, 2011]
  - Challenge: Extremities in the parameterization
  - Small spherical mis-registrations → big correspondence errors



Our Approach (Per Shape)

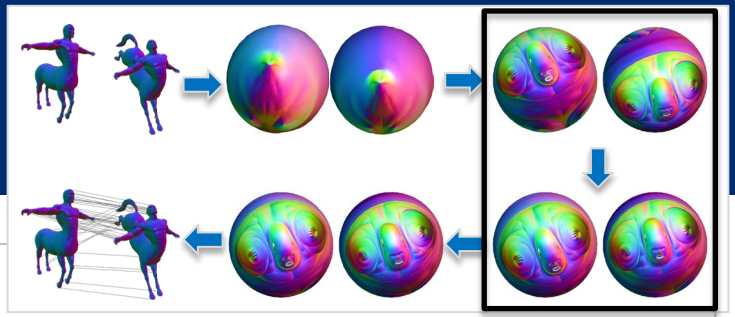


- **Authalic Evolution** [Zou *et al.*, 2011]
  - Idea: Decompress the dense regions
  - Approach: Diffuse scale factors λ to make them uniform
  - Implementation: Advect along a vector field pointing away from denser regions

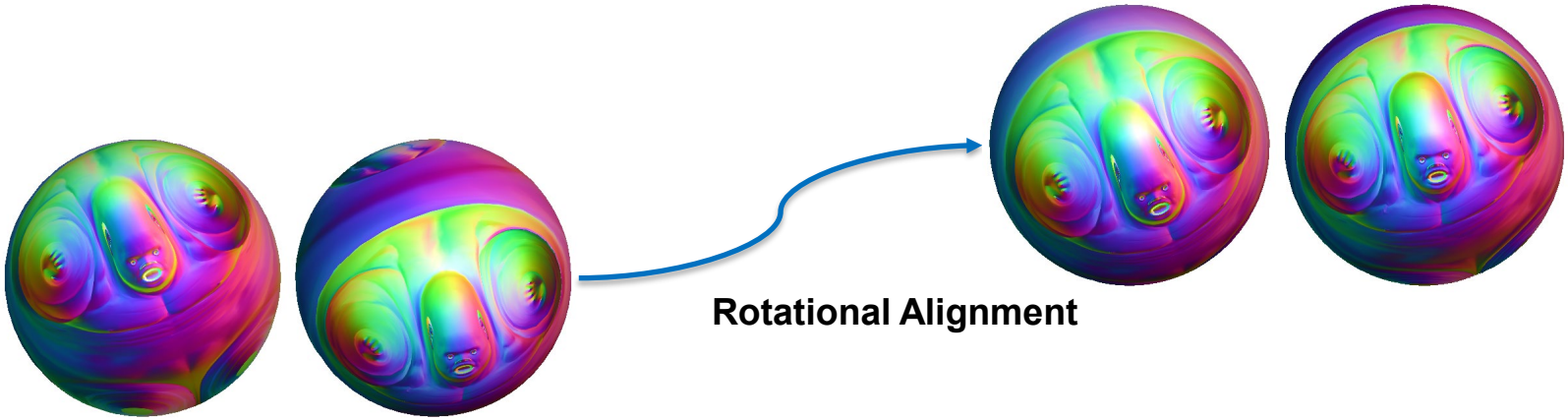


Advected Parametrizatio : $-\nabla \log \lambda$

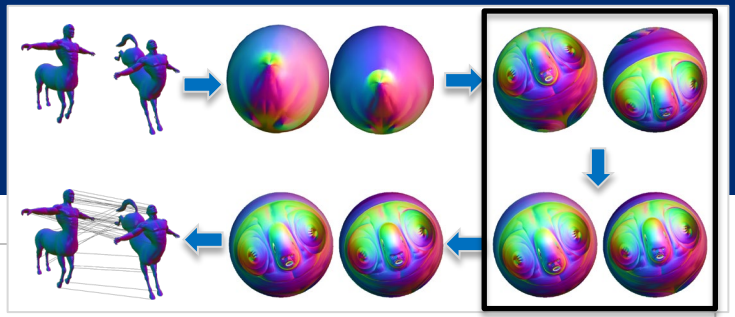
Our Approach (Per Pair of Shapes)



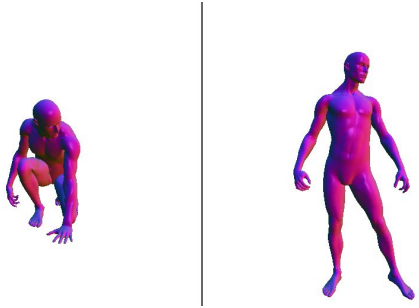
- **Rotational Alignment**
  - Find the best rotation alignment by maximizing the signals' correlation using fast signal processing [Baden *et al.*, 2018]
  - Heat Kernel Signatures [Sun *et al.*, 2009]



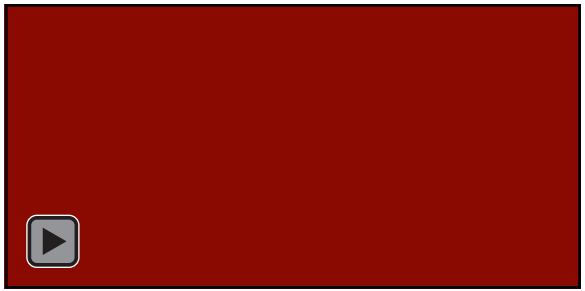
Our Approach (Per Pair of Shapes)

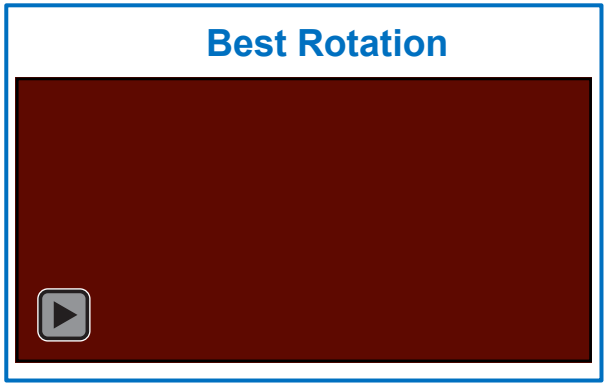


- **Rotational Alignment**
  - Near $180°$ (intrinsically) symmetric shapes
  - Compute the K best rotation candidates
  - Refine each one with optical flow
  - Return the one that best aligns the signals



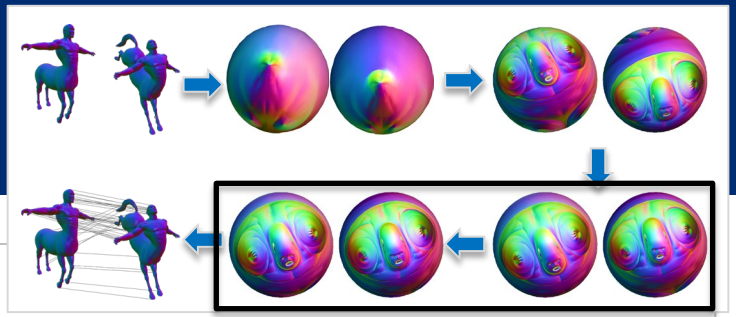
Human Shape





2 Rotation Candidates

Our Approach (Per Pair of Shapes)

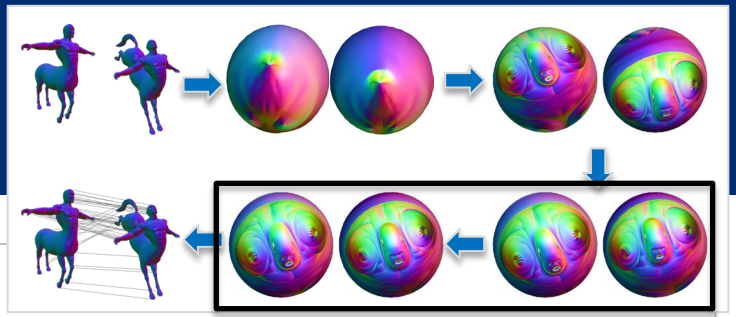


- **Optical Flow**
  - Solve for a vector field that best registers the signals [Prada *et al.*, 2016]
  - Coarse-to-fine approach

Vector Field | Advected Parameterization

Level 0

Our Approach (Per Pair of Shapes)

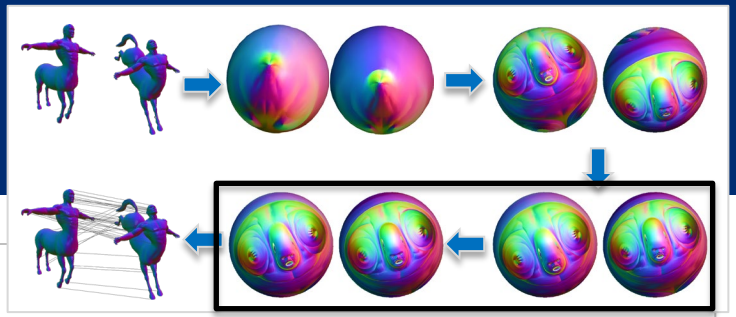


- **Optical Flow**
  - Solve for a vector field that best matches the signals [Prada *et al.*, 2016]
  - Coarse-to-fine approach

Vector Field | Advected Parameterization

Level 1

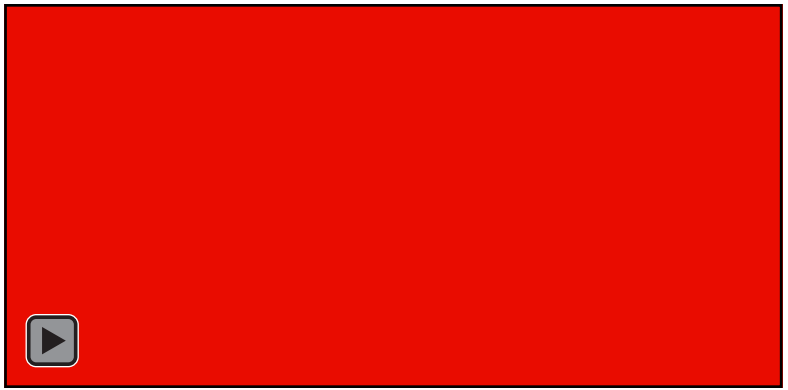
Our Approach (Per Pair of Shapes)



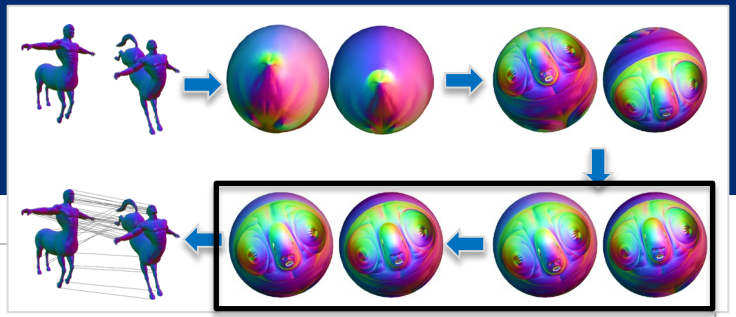
- **Optical Flow**
  - Solve for a vector field that best matches the signals [Prada *et al.*, 2016]
  - Coarse-to-fine approach

Vector Field | Advected Parameterization

Level 2



Our Approach (Per Pair of Shapes)

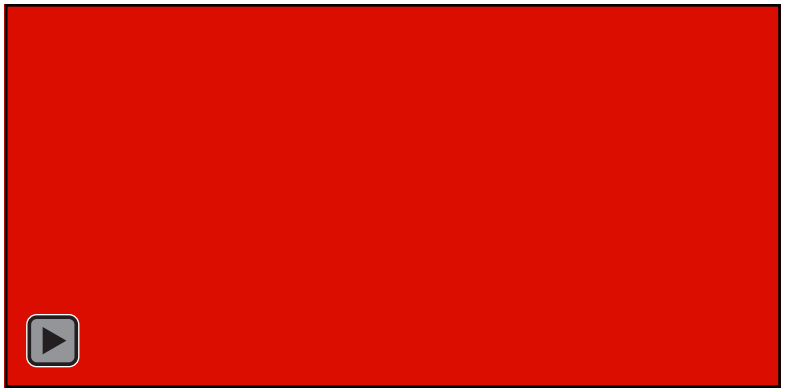


- **Optical Flow**
  - Solve for a vector field that best matches the signals [Prada *et al.*, 2016]
  - Coarse-to-fine approach

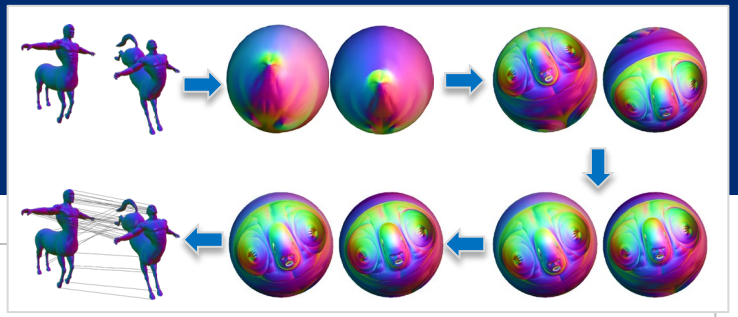
Vector Field

Advected Parameterization

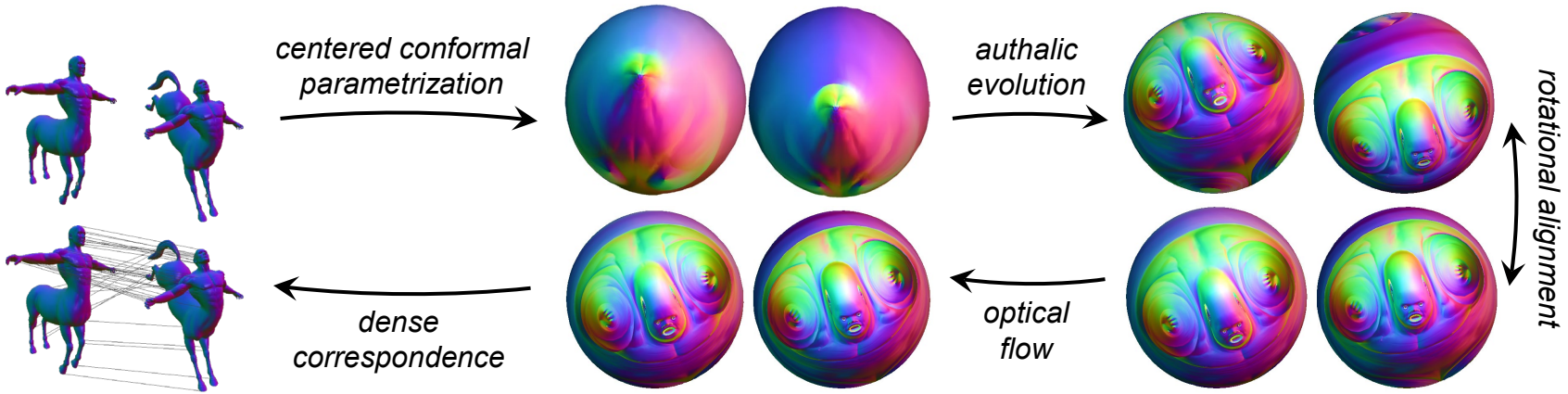
Level 3



Our Approach



- **Dense Correspondence**
  - Composition of the spherical parameterization, rotation alignment, optical flow
  - Component maps are symmetric → composition is symmetric

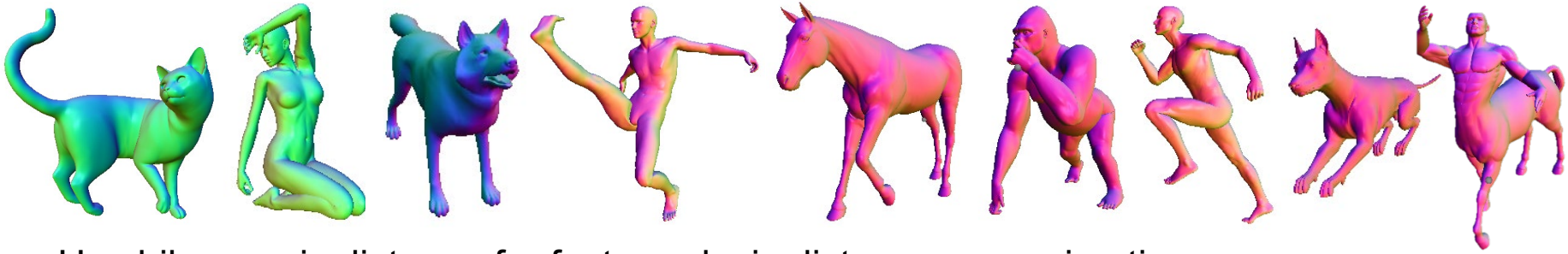


Outline

- Introduction & Related Work
- Our Approach
- Results
- Limitations & Future Work

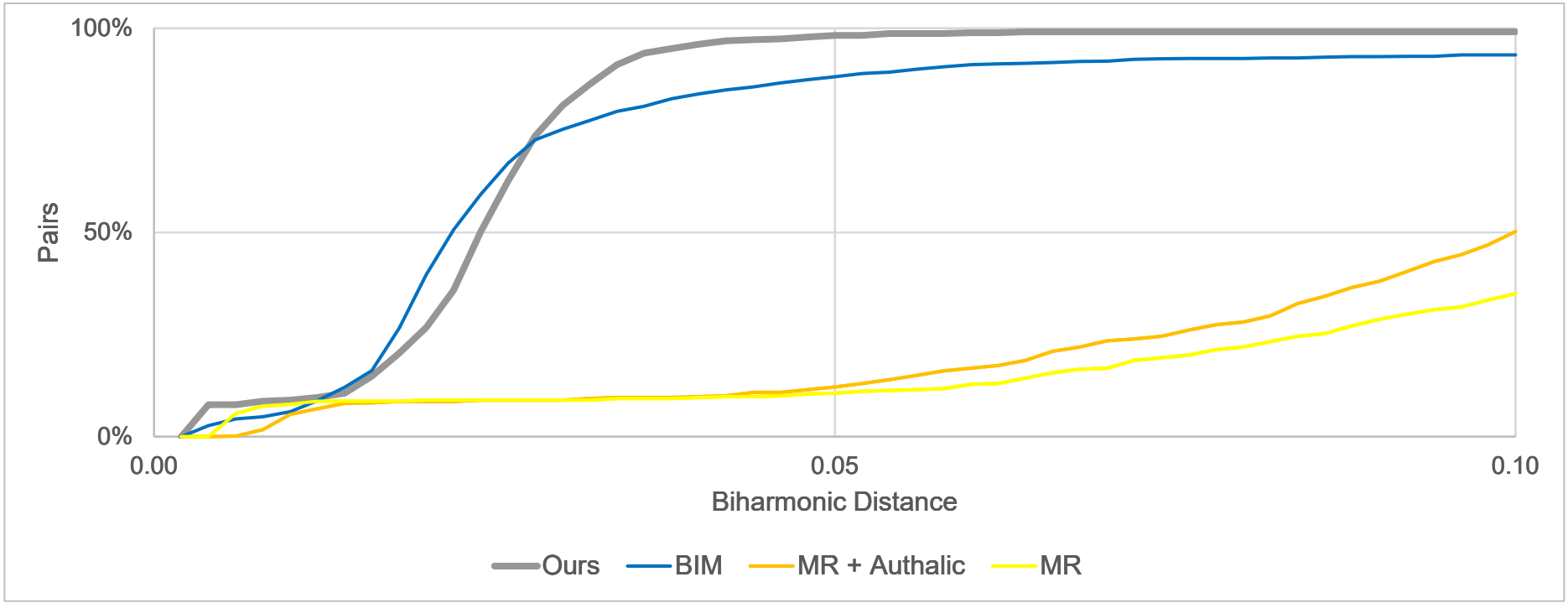
Results (1): Intra-class Correspondences

- Run on TOSCA benchmark with correspondence ground truth [Bronstein *et al.*, 2008]

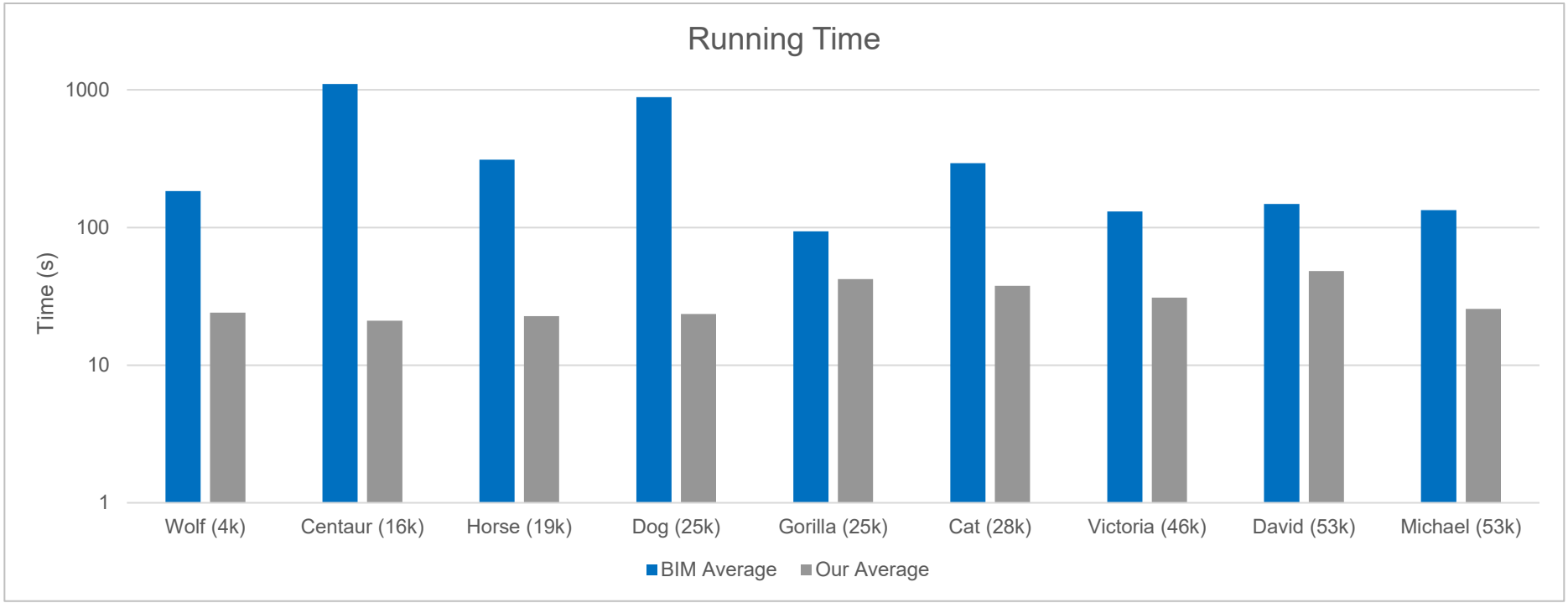


- Use biharmonic distance for fast geodesic distance approximation [Lipman *et al.*, 2010]
- Compare with
  - Blended Intrinsic Maps (BIM) [Kim *et al.,* 2011]
  - Möbius Registration (MR) [Baden *et al.*, 2018]

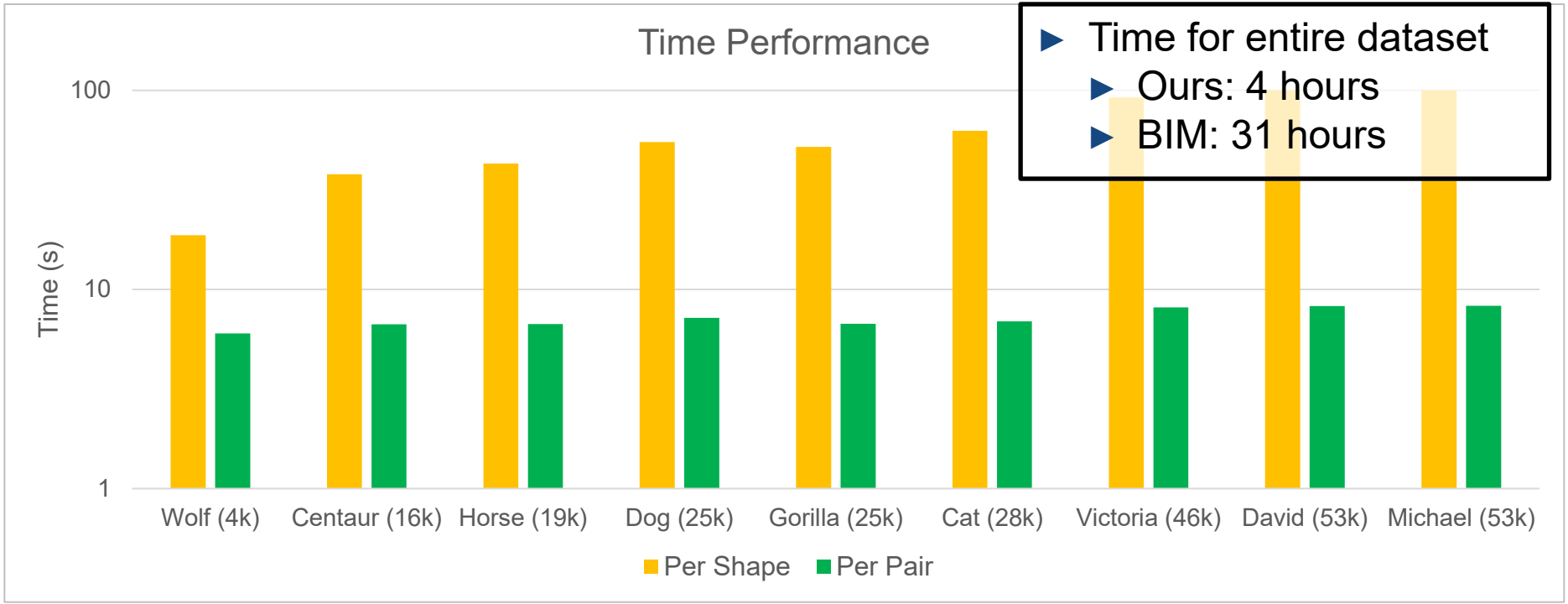
Results (1): Intra-class Correspondences



Results (1): Intra-class Correspondences



Results (1): Intra-class Correspondences



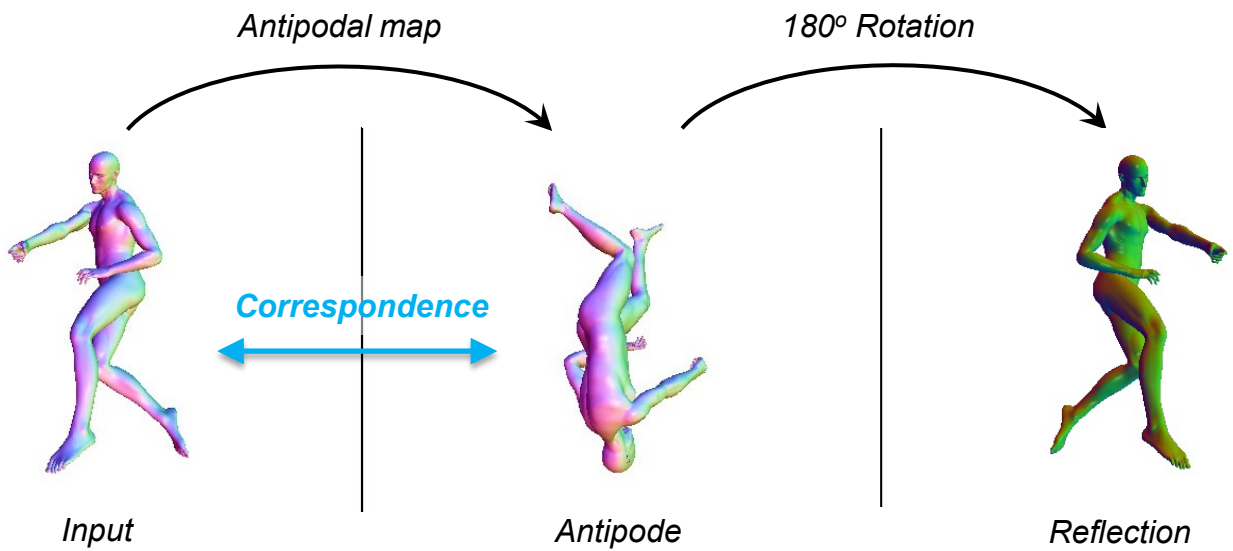
Results (2): Intrinsic Bilateral Symmetry

- Run on TOSCA benchmark [Bronstein *et al.*, 2008]
- Ground truth generated by correspondences from extrinsically bilateral symmetry

- Use geodesic distance from 100 randomly sampled vertices
- Compare with
  - Blended Intrinsic Maps (BIM) [Kim *et al.*, 2011]
  - Bijective Continuous ICP (BCICP) [Ren *et al.*, 2018]
  - Reversible Harmonic Maps (RHM) [Ezuz *et al.*, 2019]

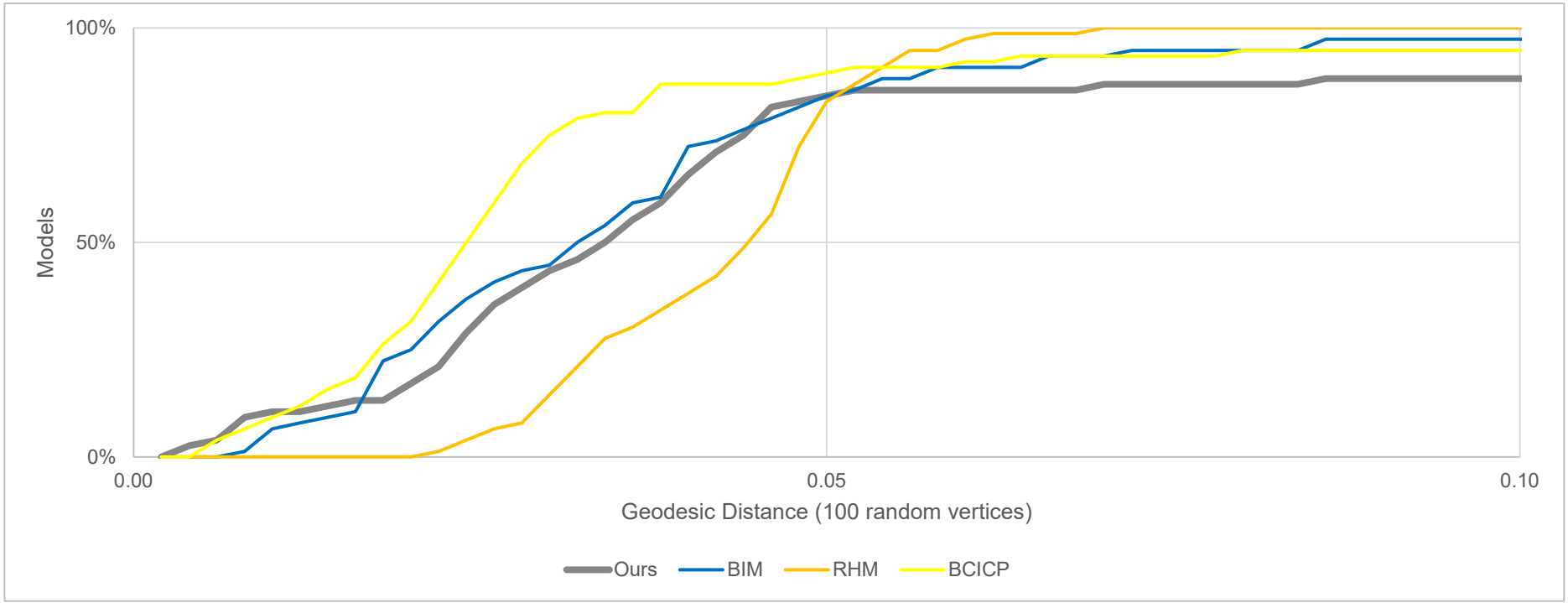
Results (2): Intrinsic Bilateral Symmetry

- Reflection = Antipodal Map + 180° Rotation



SGP 2019 - Dense Point-to-Point Correspondences Between Genus-Zero Shapes – S.C. Lee, M. Kazhdan

Results (2): Intrinsic Bilateral Symmetry



Results (2): Intrinsic Bilateral Symmetry

Ground truth | RHM | BIM | BCICP | Ours

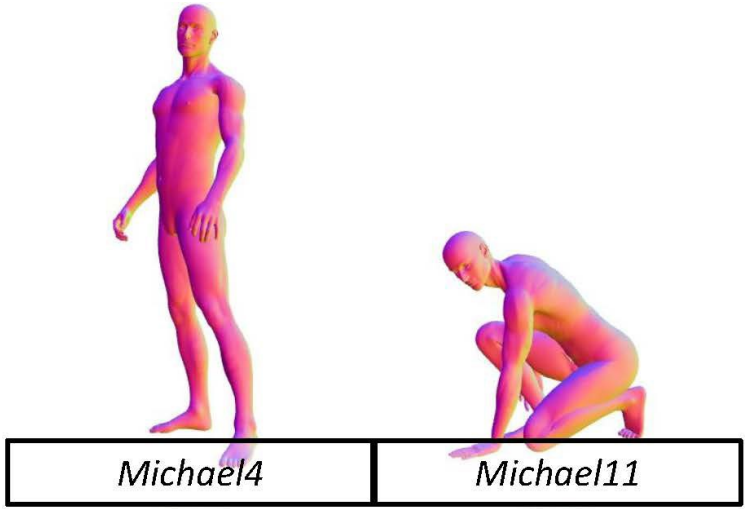
5K Model	BIM	BCICP	RHM	Ours
Per Shape	268.43s	111.25s	169.00s	43.35s

Outline

- Introduction & Related Work
- Our Approach
- Results
- Limitations & Future Work

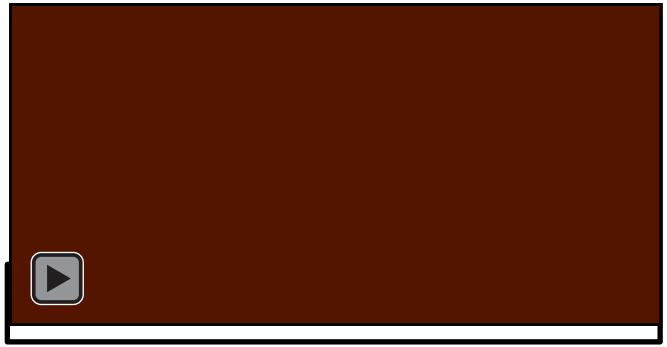
Limitation

- Poor rotation selection



Correspondence error: 8.18

Biharmonic distance: 0.03

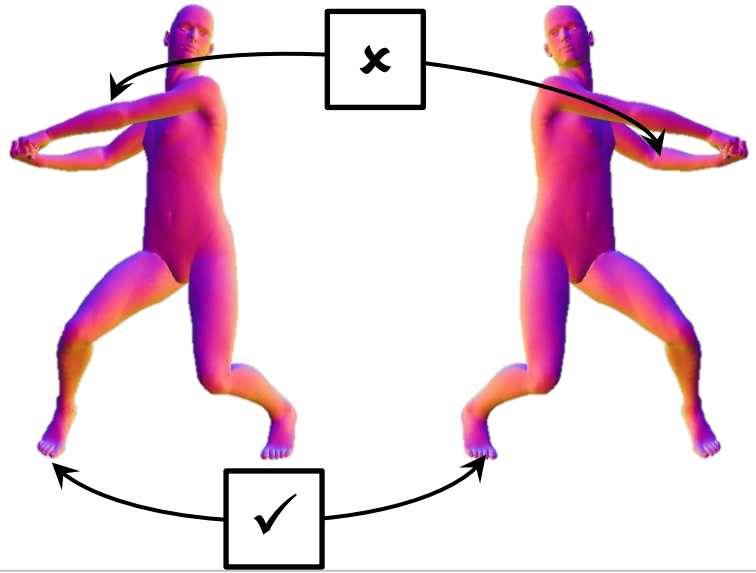


Correspondence error: 8.16

Biharmonic distance: 0.44

Limitation

- Greedy optical flow

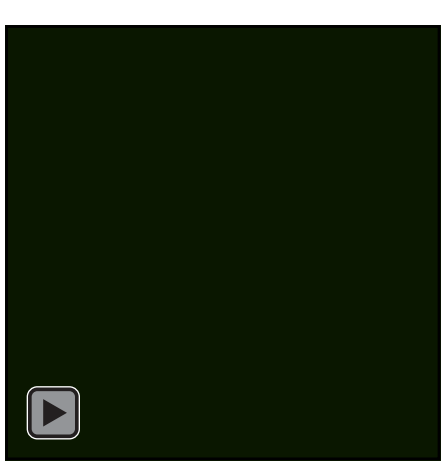
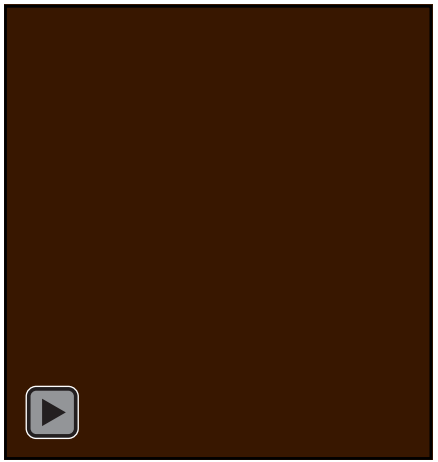
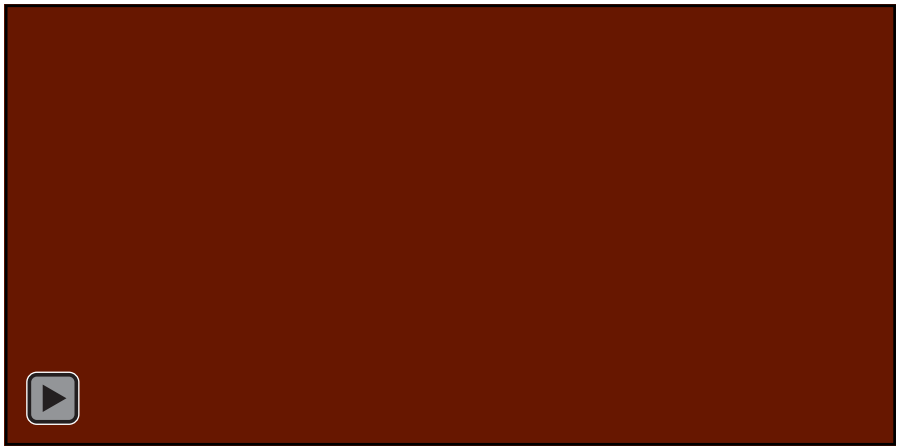


Conclusion & Future Work

- Conclusion
  - Fully automatic system for dense correspondences between genus-zero shapes
  - Competitive with most state-of-the-art methods
  - Faster

- Future Work
  - Use other signals
  - Use pull-back metric

Thank You

Source codes available at: https://github.com/mkazhdan/DenseP2PCorrespondences